Dedicated to Prof. Billy E. Rhoades on the occasion of his 90th anniversary

On Caristi's fixed point theorem in metric spaces with a graph

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ABSTRACT. We generalize the Caristi's fixed point theorem for single valued as well as multivalued mappings defined on a metric space endowed with a graph and w-distance. Particularly, we modify the concept of the (OSC)-property due to Alfuraidan and Khamsi (Alfuraidan M. R. and Khamsi, M. A., *Caristi fixed point theorem in metric spaces with graph*, Abstr. Appl. Anal., (2014) Art. ID 303484, 5.) which enable us to reformulated their stated graph theory version theorem (Theorem 3.2 in Alfuraidan M. R. and Khamsi, M. A., *Caristi fixed point theorem in metric spaces with graph*, Abstr. Appl. Anal., (2014) Art. ID 303484, 5.) to the case of w-distance. Consequently, we extend and improve some recent works concerning extension of Banach Contraction Theorem to w-distance with graph e.g. (Jachymski, J., *The contraction principle for mappings on a metric space with graph*, Proc. Amer. Math. Soc., **136** (2008), No. 4, 1359–1373; Nieto, J. J., Pouso, R. L. and Rodriguez-Lopez R., *Fixed point theorems in ordered abstract spaces*, Proc. Amer. Math. Soc., **135** (2007), 2505–2517 and Petrusel, A. and Rus, I., *Fixed point theorems in ordered L−spaces endowed with graph*, Proc. Amer, Math. Soc., **134** (2006), 411–418).

1. INTRODUCTION

Caristi's [8] fixed point theorem is considered as one of the most beautiful extensions of Banach [5] contraction theorem. The proofs given for Caristi's result vary and use different techniques (e.g. [6, 7, 19]). The original proof is based on iterates and use of transfinite induction. It is worth mentioning that because of Caristi's result close connection with Ekeland's [10] variational principle, many authors refer to it as the Caristi-Ekeland fixed point result. For some important contributions on this topic and related results, we refer the reader to see [4, 9, 12–14, 18, 20, 21, 27, 28, 31].

In 1996, Kada et. al.[16] introduced the notion of w-distance and used it primarily to generalize Caristi's fixed point theorem, Ekeland's variational principle applied to non-convex minimization problems, see Takahashi [29] for details.

Proving existence results of fixed points for monotone, order preserving mappings has been a relatively new development in metric fixed point theory. This direction was initiated by Ran and Reurings [25] (see also [30]) while investigating some applications of matrix equations. Subsequently, Nieto [22, 23] and others [11, 24] modified and improved Ran and Reurings results. In 2008, Jachymski [15] using the language of graph theory (which subsumes the partial ordering) introduced the concept of G-contraction on a metric space endowed with a graph and obtained some fixed point results which unified most of the previous results concerning partial ordering e. g. ([11, 22–25]). Recently, Alfuraidan and Khamsi [3] proved the analogue of Caristi's fixed point theorem for monotone, order preserving mapping satisfying the (OSC)-property and also stated a graph theory version theorem (Theorem 3.2 in [3]).

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Abstract

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